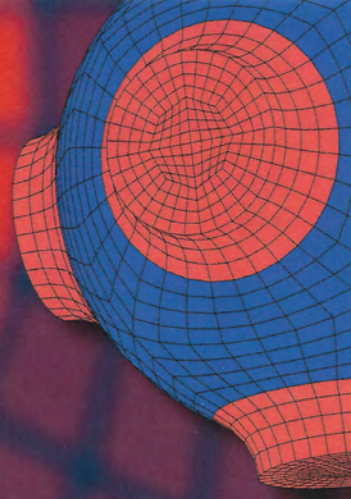


Advanced Structured Materials

Holm Altenbach  
Julius Kaplunov  
Hongbing Lu  
Masayuki Nakada *Editors*



# Advances in Mechanics of Time-Dependent Materials

 Springer


Holm Altenbach · Julius Kaplunov · Hongbing Lu ·  
Masayuki Nakada  
Editors

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## Preface

The book presents a variety of recent theoretical and experimental techniques for modelling of advanced viscoelastic materials. The book has a significant interdisciplinary flavour and reports on important achievements of the international academic community. This is a collection of selected chapters reporting on the current trends in Mechanics of Time Dependent Materials. The authors are renowned experts in the field affiliated with research intensive universities in Europe, North America and Far East.

The book covers a number of cutting-edge themes, such as characterization of linear and nonlinear mechanical behaviour of viscoelastic materials and their composites, taking into consideration finite deformations, dynamic loading, microstructure, phased transitions, along with failure and fracture phenomena. The contributions are inspired by advanced applications in modern technologies, e.g. injection moulding, extrusion etc. A variety of theoretical and experimental aspects addressed in the book will be of interest for a broad interdisciplinary audience, including but not restricted to mechanical, civil and chemical engineers, as well as material, bio and geoscientists. Postgraduate students in related areas will also find useful many of the chapters as additional literature sources.

The volume is dedicated to the 70th anniversary of Prof. Igor Emri. Prof. Emri. is an internationally leading authority in mechanics of time-dependent materials and related topics. He originated numerous influential developments in this area and co-authored around 300 publications on the subject.



Professor Emri was born in Musrka Sobota, Slovenia in 1952. He graduated from the Faculty of Mechanical Engineering at the University of Ljubljana in 1977. He was awarded a Ph.D. from California Institute of Technology, USA, in 1981. For a long time, Prof. Emri was affiliated with the University of Ljubljana. He was promoted there to a Full Professorship in 1996. Soon after, he established the Department of Mechanics of Polymers and Composites leading it until his retirement in 2016.

Professor Emri is involved in a variety of important academic activities. In particular, he is one of the founders and Editor-in-Chief of the reputable international journal »Mechanics of Time Dependent Materials«, published by Springer-Nature. His outstanding achievements received a number of major awards and honours. He is an international member of USA National Academy of Engineering (NAE) and also a member of European Academy of Sciences and Arts, European Academy of Sciences, Slovenian Academy of Sciences and Arts and several others.

Professor Emri served as the Chairman of the Science Europe Scientific Committee on Engineering and Technology (ENGITECH), the Co-Chairman of the Science Europe Scientific Advisory Committee, the President of the Society of Experimental Mechanics (SEM), and the President of the International Committee on Rheology (ICR). In 1993 I. Emri jointly with his academic mentor W. G. Knauss from the California Institute of Technology, have created the new research field called "Mechanics of Time-Dependent Materials—MTDM". MTDM was first organized as Technical Division (TD) of SEM.

Another of his key contribution to the community service is related to the organization of numerous international scientific events. In particular, he launched the series of major conferences on Mechanics of Time-Dependent Materials taking place all over the world since 1995. Fruitful less formal workshops on Advances in Experimental Mechanics regularly hosted by Emri's research group in Slovenia are also worth to be mentioned.

Research activities of Prof. Emri are mainly focused on mechanics of dissipative systems with emphasis on studying the effect of the rate of changing of thermo-mechanical boundary conditions on processes of structure formation of polymeric materials and their macro-, micro- and nanocomposites, as well as on the behaviour of solid granular systems. He has developed a new nonlinear viscoelastic constitutive

model, known as the so-called Knauss-Emri model, which enables modelling of nonlinear behaviour of engineering polymers and composites subject to complex time-varying thermo-mechanical loading, and prediction of their long-time behavior (durability of polymer based products and structures).

Professor Emri and his group have shown that macroscopic properties of polymeric materials and their composites, can be controlled and modified either by changing material initial kinetics, i.e. molecular mass distribution and topology of molecules, or by varying thermo-mechanical conditions, including pressure and temperature, or by high rate mechanical loading.

The pioneering collaborative efforts by I. Emri together with W. G. Knauss and N. W. Tschoegl from the California Institute of Technologies resulted in an innovative theoretical-experimental approach for analysing the structure formation processes of multimodal polyamide materials under the influence of complex thermomechanical conditions. Emri's findings were patented and implemented by BASF in the production of polyamides. Later it was found that by using these materials one can manufacture osseointegrable implants with a gradient structure that mimics properties of bones and may be used in dental and orthopaedic surgery.

Among more recent Emri's scientific results there is the development of sound insulation structures involving granular materials. The underlying theory is based on the mechanism of a "force-network" formation. His invention has a substantial potential to be implemented in various modern industries including mechanical, automotive, electrical, aerospace, railway, naval and civil engineering. This piece of work has been also patented.

The list of a few selected publications by Prof. Emri illustrating a broad range of his research interests is given below.

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